TRANSFORMATION PROBLEM :
WHAT ARE EQUATIONS ABOUT ?

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According to the devastating critics by Fujimori [1985] to my paper [1982] on transformation problem, "my" solution "cannot be said to be an extension of the transformation theory and hence cannot be said to replace the accepted one".

The argument of the author consists first in proving that my solution is just a different choice of normalizing equations. Then he criticizes my choice or the equations. To begin my reply, I shall recall the content of my paper. Then I come back to Fujimori's critics.

But first I should outline that this solution is not "mine", but a result of Friendly exchanges with D.K. FOLEY [1982] and G. DUMENIL [1980]. It is more and more accepted, and it proved to be easier to handle in joint production problems, even with rent (LIPIETZ [1979]), or discussions about the OKISHIO theorem on the tendency of profit rate to fall (LIPIETZ [1986]). Moreover, it is a piece of a more general concern on what prices are about, and more precisely the price of the labour force (LIPIETZ [1983]).

I –-WHAT DIO I TRY TO EXPLAIN?

Contrary to what FUJIMORI says (and contrary to the wishes of DUMENIL [1984]), I *do not* reject, in any of my related texts, the classical Morishima type solution. On the contrary, the paper 1 19823 (for instance) explains the following.

a. There are in Marx *two* definitions of the "value of labour force" v:

    A : it is the value of the bundle f that the worker buy with his/her wage.
    B : it is the share of value-added represented by the money of the wage (independently of the choice of a definite bundle)

b.   Morishima's solution chooses the definition A: $v = w.f.$

c.   An economic reflexion on solution A makes disappear a lot of false contradictions between it and Marx's propositions. That is :

c-1 The sum of profit may not be the sum of surplus-value, but anyway, the sum of values of what the capitalists buy with their profits (a definition consistent with definition A of the value of labour force) is equal to surplus value.

c- 2 The "independence" of the rate of profit, hence of production prices, From the structure of output, in solution A, is limited by a "compensation theorem". This is an important point (though Fujimori does not think so), for the growing importance of sectors with high composition of capital is a central point of Marx's theory of the tendency of the rate of profit to fall. It should be outlined that these two results are carried about through "duality" considerations on the "production equilibrium side", which Fujimori asserts me to ignore.

c-3 Samuelson's critics, according to which, in A-type solutions, there is no need For a labour-value and an exploitation theory, is irrelevant. As soon as one introduces L to denote both a quantity of commodity manpower" (e.g. in accounting the wage mass as ω L) and a quantity of value added (e.g. in writting the system of values $w = w\,A + L$), then one assumes a tensor (matrix of coefficients) mapping a vector of commodities manpower M into the vector of value-added L. This "tensor of exploitation" T is dependant on the length and intensity (ε, λ) of the use of one worker's day. By an appropriate choice of units of time and value, this tensor could be made equal to identity, but this must not offset the preexisting theory of labour value and exploitation to a theory of prices of production.

Hence, far from ignoring the marxist distinction between labour and labour-power, as Fujimori says, I improve Morishima's solution (and defend it against Samuelson) by introducing a sharp distinction between M and L.

d. There is a new (Duménil-Foley-Lipietz) solution.

It is based on definition B for the value of labour force. We shall come to it later. But in my texts I outline that anyway the "manpower commodity" is not produced as a "cow" or a slave. First because a lot of labour, socially needed to reproduce it, is not included in its price (the domestic labour of women). Second, because the worker does not receive a "voucher for a bundle" (and certainly not the bundle itself, as the slave does), but money. With that money he/she behaves as a free agent, even if there exists a "fuzzy bundle" of necessary needs that the wage ω is to cover. Third, because, even if we accept that these reproduction needs can be reduced to a bundle f of commodities, the price of the commodity "manpower" is computed differently from others:

$$\omega = p.f$$
and not: $\omega = (1 + r) \, p.f.$

This is sufficient to prove that, contrary to what Fujimori writes, labour power is certainly "commoditized", but certainly not "reproduced as a commodity". Marx spent a lot of bottles of ink to prove that, contrary to the bourgeois - XIXth century - civil code, the wage-relation is not simply a commodity-exchange.

e. The two solutions must be used together, and there is a "duality" between the two. Moreover, this duality is based on a dialectics about how the value of labour-force is determined. In this dia1ectics, the main momentum shifted historically from A to B. In the post world-war II period of "fordist" regime of accumulation (LIPIETZ [1985]), B is the operationnal definition, and it must be used to analyse the fall of the rate of profit in the late 60's (LIPIETZ [1986]).

That was the content of [1982]. Now let us look at Fujimori 's critics of point d.

II – SHOULD WE GIVE AN ECONOMIC MEANING TO OUR MATHEMATICAL GAMES?

Fujimori starts his critics of solution B by proposing a solution C which he judges just as good, since there are also enough equations for the variables. The particularities of C are the following:

– sum of values equals to sum of prices (weighted by gross output structure) .
– sum of profits equals to sum of surplus-values (same weighting).

Then, of course, once v (value of labour force) is choosed, even without any definition, mathematically the other variables are known. For instance, in Fujimori's solution, we have: $\omega = (v \mu L - \pi p A) x / L x$

Funny and interesting. The mathematician is happy. But the economist may object (fn 1).

So let us take the system B, step by step, and let us try, for once, to understand what we are writing. And let us accept Fujimori's order.

(1) $p = (1 + \pi) (p A + w L)$

It is just the *expression* of the equalization of rates of profit, in the "price/quantities of commodities" world. Please note that here L is a vector of commodities "manpower".

(2) $p y = L x$ (y = net product, x = output).

This is a *choice* of numeraire. The unit of money is such that the value of net product of the period is equal to the value added of the period. Two comments :

– It is a nice choice, since we are to "see" the redistribution of value added accross the product of the *same* period, according to non-homogenity of organic composition of sectors.

– Here L means a quantity of labour considered as a quantity of labour-value. We are in the "value" world, contrary to what Fujimori writes. Of course, in equation (1), L stands for M with the tensor T normalized to 1 through the choice of time and value units.

(3) $v (1 + \mu) = 1$

This is a *definition* of μ (rate of surplus-value)

(4) $y = x - A x$

This is a *definition* of net product.

(5) $v = \omega$

There is the rub. For Fujimori, it is a new and ad hoe hypothesis ("suppose that the value of labour force is equal to real wage") . For me it is a straitforward *expression of definition* B for v (this is may be more exp1icit in (LIPIETZ [1983]). Since Fujimori likes counting equations, let us develop:

(5') $v = \omega.u$

This is *Definition* B of v. It is consistent with unnumerable statements of Marx M DUMENIL [1980]) such as "the worker receives in money four hours out the height he works".

(5") $u = p y / L x$

This is the only possible *definition* of the value of money: the ratio of all the value added in a period, wich is exactly embodied in the net product, to the total price of this net product of the period. Any other definition (through the conditions of production of gold ? through a peculiar exchange ratio between money and one peculiar commodity?) is unacceptable.

Now, from (2), (5'), (5"), we have u =.1 and v = ω (5).

(6) w = w A + L

This is the expression of the marxist labour-theory of value.

From that 2n + 4 - equations system follows:

(7) r is unique, depends on {μ, A,L, x}, that is: exploitation, composition of capital in sectors, weighting of sectors, as in Marx.

(8) μ (p A + ω L) x = v μ L x, sums of profits : sum of surplus value.

Hence (8) is a theorem stemming from one normalisation (2) and one definition for the value of labour force (expressed by (5)). Now, if one changes the normalisation (2), choosing the yen as numéraire, one will find, through national account, that:

u = k (in hour per yen)

and :⎧ sum of surplus value = k. sum of profits
     ⎩ v = k.ω

More generally, we may give up any normalization (2), then the theorem (8) will be: "The ratio of sum of profits to sum of surplus value is equal to the ratio of sum of price to sum of surplus-value" (Fn.2).

On the contrary, (5) expresses the fact that the rate of exploitation μ is known *before* any peculiar choice of bundle by any worker. So workers may choose bundles of different values, without anyone being "more exploited" than another. A difficulty of solution A that led to

paradoxes (STEEDMAN [1977]) such as imposing *two* "budget constraint", one in value, one in price, on workers' behaviour (ROEMER [1978]).

This does not mean that solution B forget "reproduction" and puts ahead "distribution". It only assumes that v is sufficient to buy the means of reproduction. When solution A expresses a fixed bundle of uses of wage, it *also* assumies that this bundle is a "good" reproducing bundle.

If one keep in mind that 1 insisted, contrary to Morishima's standard solution, on the importance of conditions of exploitation (T, ε, λ), it is difficult to characterise, as Fujimori does, my approach as "distributional".

CONCLUDING REMARKS

It is fascinating to discover that economics, even marxist economics, can be handled mathematically, just as physics. But physicians are used to be carefull, and draw sharp distinctions between definitions of variables, choice of units ("normalization") and equations expressing the conceptual content of their, theory. From any subset of these equations, new equations appear as theorems. Mathematically, the new equations could have been choosed as hypothesis, and some initial equations as theorem. But this formal game is of no meaning to the physician.

Nor is it to the economist. We have to draw distinctions between the equations expressing the content of our basic theory (labour theory of value: (6), exploitation: (5'), equalization of profit rates: (1)) , the *definitions* of magnitude (as in (3), (4) (5")), and sometimes a usefull choice of units (as in (2)).

But it may be better not to choose "too good" units, and put the result in a shape that help to keep in mind the meaning of equations. A lot of ink could have been saved on transformation problem if equations had been written inducing explicitly (the tensor of exploitation) and u (the value represented by the unit of money).

Alain LIPIETZ

NOTES

1) That "teratological" result stems from the unfortunate choice of the weighting in "sum of prices". More one this in footnote 2.

2) Yet the definition of "sum" (i.e. the wieghting) remains important. If we accept, unwisely, "p.x = v.x" at level (2), then we shall get another teratological result at level (8) . But in that case the oddity is meaningfull. It could straighforwardly be expressed: "The smitham sum of profits equals the smithian sum of surplus values". "Smithian sum" means the sum of all terms, including the ones embodied in constant capital at every vintage in the past. (see LIPIETZ [1982, 1983]).

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