But can we reduce labor power to an output requiring as input a vector d with a cost price $\Sigma p_i d^i$? Such a formulation would amount in fact to a model of a "quasi-slave" mode of production, that is, a mode of production where slavery exists within the units of production, but relations between units of production are commodity relations (as was the case in the Southern states of the U.S. before the Civil War).[14] But in fact workers in developed capitalist society receive a money wage which they can choose "voluntarily" to spend according to their needs. Certainly the general standard of living limits their choice to a certain fuzzy set of bundles, and the value of bundles in that fuzzy set at a given time in the history of class struggle is a basis for the value of labor power. But this does not justify our adopting a scheme of unilateral causality like the one shown in Fig. 2. It is much more accurate to think of matters as in Fig. 3, with a "loop" between d and e.

But here a problem arises. The budget constraint of workers in reality is expressed in the same system of prices as the commodities they buy. But in Morishima-type solutions the budget constraint is expressed in values, while the system of prices depends of the bundle of commodities finally chosen. This difficulty is resolved only in form by Roemer's use of the fixed point theorem [16].

On the other hand, matters are much simplified if we interpret w as a "quantity of paid labor," a share $1/(1+e)$ of the value added, the money equivalent of which is to be spent at prices of production to meet socially determined needs. The prices of production then should not depend on d, but, as in Marx, on y. The result is a bundle of consumption goods d whose value in terms of labor embodied (though not in terms of money, that is abstract social labor) could well be different from w. This pattern of consumption d, if it does not satisfy the workers, could then become the basis for a renegotiation of the proportion e of the value added received by the workers.

This solution, which requires us to treat constant and variable capital differently, turns out to be consistent with the ideas Marx left concerning the transformation problem. This is obviously not an argument for its correctness, but insofar as students of the transformation problem have tried to formalize Marx's model in order to show that it is inconsistent, it is important to make sure that they have in fact formalized Marx's model, and not somebody else's. What does Màrx tell us, after he has himself criticized his clumsy little model of the transformation? That it is necessary to transform the elements of cost prices as follows:

[14] In any case, labor power could not be identified (as in Samuelson) with the product of sector 0, since, in the enterprises of that sector (the households) labor (the wife's) would be unpaid and the "boss" would sell the product without receiving any profit. It might be otherwise if workers lived in boarding houses run by capitalists, but then the price of labor power would be $(1+r)p \cdot d$, not $p \cdot d$.

> So far as the constant portion is concerned, it is itself equal to the cost-price plus the surplus-value, here therefore equal to cost-price plus profit, and this profit may again be greater or smaller than the surplus-value for which it stands. As for the variable capital, the average daily wage is indeed always equal to the value produced in the number of hours the labourer must work to produce the necessities of life. But this number of hours is in his turn obscured by the deviation of the prices of production of the necessities of life from their value. [12, Vol. III, p. 161]

In other words Marx thought that in the case of constant capital it was necessary to transform the value of commodities. But in the case of variable capital, the wage, insofar as it represents a share of the value added, that is, as a "number of hours," is conserved by the transformation, while the labor time itself, considered as the equivalent of a particular bundle of commodities, is transformed. This is exactly what happens in the solution just proposed.

It remains to check up that this solution is mathematically consistent.

2. *The Model*

Suppose v is the vector of the values of the commodities, determined by A and ℓ. Suppose we have a given rate of exploitation e and a value of labor power w with $w = 1/(1+e)$. We seek a redistribution of the total value added (that is, of the global flow of abstract labor) produced in the period, over the *net* product y of that same period.

Here we meet a second criticism of the currently accepted solutions to the transformation problem. For the most part, this work tries to verify the "Marxian equalities" of the type "sum of prices equals sum of values," and so on, without posing the question: "sum of which prices?" Marx observed that we cannot aggregate all prices or all profits and hope to reach the sum of values or of surplus value:

> In applying this approach to the aggregate product of society, we must make some rectifications. Looking upon society as a whole, the profit contained in, say, the price of flax cannot appear twice—not both as a portion of the linen price and as the profit of the flax. [12, Vol. III, p. 160]

Today, thanks in part to the work of Marx in Volume II of *Capital*, written after Volume III, on reproduction, this "rectification" is easy to make, since we understand the relation between gross and net product.

Thus the redistribution of value added is indeed a redistribution, in that it reallocates a constant quantity of the substance abstract labor. Let us call p_i^* the value reallocated to each commodity i. The vector p^* of reallocated values defines the system of relative prices of production (the level of prices depending on the choice of the numeraire). We must have, by definition

$$p^* \cdot y = v \cdot y. \qquad \text{(H1)}$$

On the other hand, this reallocation ought to achieve an equalization of profit rates on invested capital, so that the value redistributed to good i ought to be equal to $(1+r)$ times the sum of constant capital (evaluated in redistributed values) and of variable capital (measured by the value paid to workers in exchange for their giving over their labor power), $(1+r)$ being the same in all sectors. Then, if we assume as we have all along that the choice of units of quantities of labor power and of abstract labor expended are such that the linear transformation T may remain implicit, we must have (as a definition, the consistency of which with the earlier condition must be checked):

$$p^* = (1+r)(pA + w\ell). \tag{H2}$$

We can define this redistribution in another way. We want the "labor commanded"[15] directly (during the last period, period 0), and indirectly (in earlier periods 1, 2,...) to produce good j[16] to contribute to the redistributed value p^* of the product in the proportions $(1+r)^{n+1}$.

$$p = (1+r)w\sum_{0}^{\infty}((1+r)^n\,\ell A^n). \tag{H2$'$}$$

Equation (H2′) is equivalent to (H2), which can be written as

$$p = w\ell(I/(1+r) - A)^{-1}, \tag{H2$''$}$$

which gives (H2′) when we expand the inverse matrix as a series.[17]

[15] "Labor commanded" is a bastard concept of Classical Political Economy (Smith and Ricardo), which Marx uses explicitly in *Theories of Surplus Value* and implicitly in Volume III of *Capital*. There it designates the value or price of purchased manpower when used as an index of labor expended. In other words, it is variable capital V in so far as it can serve as an index of $V+S$, that is, in so far as we can hold w and the transformation T constant. For instance, when Marx denotes the organic composition of capital as C/V in Volume III he remarks explicitly that V is used as an index of embodied labor. Hence the difficulty many writers have had who do not see why the rise in the organic composition of capital eventually involves a fall in the rate of profit. If we understand (as Marx did) that a rise in the organic composition of capital is in fact an increase in $C/(V+S)$, we can easily see that the rate of profit must tend uniformly to zero with the growth of the organic composition of capital, whatever may happen to $e = S/V$. (On "labor commanded" and its traps, see [4, 9].)

[16] The quantity of abstract labor embodied directly in j is ℓ_j; the quantity directly embodied in its means of production is $\ell \cdot A_j$; in the means of production of the means of production $\ell \cdot [A^2]_j$; and so forth. Since T is the unity tensor, one gets "labor commanded" by multiplying by w.

[17] The series expressed in (H2) will converge to the inverse matrix as long as $(1+r) < 1+R$, where $1/(1+R)$ is the dominant eigenvalue of the matrix A. This condition will hold, as we shall see, as long as workers consume something, since the matrix A is productive. (See [15]).

Notice that the same operation can be applied to

$$v = (1+e)\, w\ell(I-A)^{-1},$$

which yields

$$v = (1+e)\, w \left(\sum_0^{\infty} \ell A^n\right).$$

Now we can see that redistribution consists in reallocating the whole value[18] in such a way that the surplus value is proportional, not to the simple sum of the labor commanded in earlier periods, but to the sum of labor weighted by a factor $(1+r)^{n+1}$. Samuelson's analogy to the case of a value added tax and a turnover tax is correct, but not by itself enough, since the "turnover" concept is not well defined. Suppose a weaving firm buys a spinning firm. The labor of the spinners, which before appeared as constant capital in the weaving sector (according to (H2)) now appears as variable capital, although it is tied up for two periods instead of one. Equation (H2′) correctly handles this situation.

The system of redistributed value which we seek (and from which we can derive the whole system of prices of production once the numeraire is given) is now well defined by (H1) and (H2). The questions remain whether such a system exists, and if it does exist, what properties it has. We can prove the following theorem, which sums up Marx's conclusions concerning the transformation problem, quite easily.

THE MARXIST TRANSFORMATION THEOREM. 1. *For every structure of output, there exists one and only one capitalist redistribution of value which equalizes rates of profit.*

2. *If we choose the numeraire so that the sum of value added in value terms is equal to the sum of prices of net product, then the sum of profits is equal to the sum of surplus value.*

3. *The average rate of profit is a function of the rate of exploitation, of the technical coefficients of production in each sector, and of the allocation of social labor among the sectors, and thus of the structure of output.*

Proof. 1. For all y, there exists $(p^*, (1+r))$ with

$$p^* \cdot y = v \cdot y, \tag{H1}$$

$$p^* = (1+r)(pA + w\ell). \tag{H2}$$

[18] This decomposition of the "constant capital," into variable capital and surplus value (or, transformed, of the price of intermediate goods into wages and profit) was already known to Adam Smith.

In fact, when we write (H2) as (H2′), we see that p^* is a continuous increasing function of $(1+r)$, so that $p^* \cdot y$ is a continuous increasing function of $(1+r)$ as well.

When $r = 0$, $p^* \cdot y = v \cdot y/(1+e) < v \cdot y$.

As r approaches the maximum profit rate in (H2′), $p^* \cdot y$ tends to infinity (see footnote 17). Thus there exists one and only one value of r and one and only one vector p^*, which satisfies (H1). The critical rate r must be positive[19] and depends on y.

2. Let y be the net product (in the sense of national income accounting) which embodies the value added, and $\mathscr{Y}$ the corresponding vector of gross outputs. By definition:

$$p^* \cdot y = v \cdot y \qquad \text{and} \qquad \mathscr{Y}(I - A) = y.$$

We have that

$$\begin{aligned} \text{the sum of profits} &= p^* \cdot y - w\ell \cdot \mathscr{Y} \\ &= v(I-A)\mathscr{Y} - w\ell \cdot \mathscr{Y} \\ &= ew\ell \cdot \mathscr{Y}, \end{aligned}$$

which is the sum of the surplus values.

3. $r =$ sum of profits/invested capital

$$= ew\ell \cdot \mathscr{Y}/(p^*A + w\ell) \cdot \mathscr{Y}.$$

Thus

$$r = e/((p^*A\mathscr{Y}/w\ell\mathscr{Y}) + 1)$$

and

$$p^*A\mathscr{Y}/w\ell \cdot \mathscr{Y} = \sum (p^* \cdot A_j/wl_j)(l_j Y_j/\ell \cdot \mathscr{Y}).$$

This is the average of the organic compositions of capital in the various sectors, each weighted by the share of wage labor employed in that sector, evaluated at prices of production. These organic compositions are functions of the technical composition of capital in the sectors, and of the system of

[19] We can see that for r to be positive, it is necessary and sufficient that w be less than 1, or that e be positive. This establishes in this model the "Fundamental Marxian Theorem" of Morishima and Okishio.

prices of production which itself depends only on e, y, and the technical coefficients a_j^i and l_j.[20]

This last formula has the defect that it presupposes a knowledge of p^*. I have written it because of its simularity to Marx's formula, which Marx himself recognized to be only a first approximation.[21]

We can compute r as a direct function of the basic parameters, using the method Dumenil and Roy employed to study the Morishima-type solution.

We start with $p^* \cdot y = v \cdot y$.

Using (H2″) and the fact that $w(1+e) = 1$, we get

$$w\ell(I/(1+r) - A)^{-1} y = (1+e)\, wv \cdot y$$

and

$$e = \ell(I/(1+r) - A)^{-1} y / v \cdot y - 1.$$

This rate depends only on the structure of output y. Let y^* be parallel to y with $v \cdot y^* = 1$, and we have

$$e = \ell(I/(1+r) - A)^{-1} y^* - v \cdot y^*$$

Since

$$\ell = v(I - A),$$

we get

$$\begin{aligned} e &= v((I-A)(I/(1+r) - A)^{-1} - I)\, y^* \\ &= v((rI/(1+r) + I/(1+r) - A)(I/(1+r) - A)^{-1} - I)\, y^*, \\ e &= rv(I - (1+r)A)^{-1}\, y^*. \end{aligned}$$

e is thus a continuous increasing function of r, so that we can invert it to get $r = f(y^*, e)$.

This function takes the place of the relation r: $f(d^*, e)$ of the last section. The structure of output y^* takes the place of the structure of workers' consumption d^*. The curve relating r to e is increasing, concave, has an

[20] Of course these "technical" coefficients a_j^i, l_j are the material expression of social relations: the specialization of labor, taylorism, fordism, and so on. By "technical" I mean only that these coefficients are given before the conditions of appropriation of surplus value, before T, e, w are given, and therefore (and here is an essential difference from the Morishima-type of solution) completely independently of d.

[21] In Marx's approximation, which is $r = \Sigma S_i / \Sigma(C_i + V_i)$, the organic compositions of capital are evaluated in the labor values. This is the *only* consequence of Marx's use of this approximation.

asymptote at $r=R$, where $1/(1+R)$ is the dominant eigenvalue of A, and a finite slope at the origin.[22] The envelope of the family indexed by y^* is composed, as in Section II.4, of segments of particular curves (the ones corresponding to the y^* which maximize and minimize the organic composition of capital at a given r).

IV. Comparison of the Two Solutions

We have now reached, in the new solution (which I will call "system B" from now on), nearly all the conclusions Marx expected from the transformation of values into prices of production. Are the conclusions drawn from Morishima-type solutions (from now on "system A") false? Of course not: They are mathematically correct, and economically compatible with the marxist theory of value and of exploitation. Suppose we have made the transformation for given e and w, according to method B. We now know p and r. For this system of prices, this wage, and this rate of profit, suppose that workers choose a bundle of goods d. Then using bundle d in method A, we must arrive at the same p and r. Nevertheless, we know that for given e or w, the two solutions A and B do not give the same results: One violates the marxian equality of profits and surplus value, the other does not.

There is no contradiction here: The simple point is that e and w do not have the same meaning, nor the same quantitative measure, in the two systems, though they serve as indices to represent the same theoretical concepts.

In system A, $w(A)$ is the value of what workers consume:

$$w(A)=v\cdot d.$$

In system B, $w(B)$ is the part of the value, which workers have created, to which they have received a claim in the form of wages to spend on the market where prices are regulated by the redistributed value.

$$w(B)=1/(1+e(B)).$$

There is no reason to expect that $w(A)=w(B)$, and that $e(A)=e(B)$. In fact, in general, the value embodied in the uses of wages is not equal to the part of the produced value paid to the workers and spent by them according to the system of redistributed value.

$$w(A)=v\cdot d\neq w(B)=1/(1+e(B)).$$

[22] If we work out the series expression for $e=f(y^*,r)$, it is obvious that, for all r, d^2e/dr^2 and de/dr are positive and that, at the origin, e/r tends to $v(I-A)^{-1}y^*$.

Once the transformation has taken place, for a given y, according to system B, does there exist a structure of workers' consumption d^* such that $w(B) = v \cdot d^*$? The answer is clear from the analysis of Section II-2 of this paper. The required d must be chosen so that y is the right eigenvalue of the matrix $M = A + d \times \ell$. (We shall not discuss here in what interval this inverse mapping is one-to-one and continuous). In every other case, all the results hold good separately, but $e(A) \neq e(B)$ and $e(A) = f(y^*, d^*, e(B))$, where $f(y^*, d^*, \cdot)$ is the composition of the inverse of $f(d^*, \cdot)$ and $f(y^*, \cdot)$. These results are summed up in Table I.

Solution B is not only closer to the intuitions in Marx, but it is also much easier to manipulate mathematically. Must we then relegate the "old" solution, the fruit of a half-century's work, to the museum of curiosities in

TABLE I

Comparison of the Two Solutions

System A	System B
v = vector of labor values = labor time embodied in commodities	
e = division of embodied labor time between paid and unpaid labor.	
$e(A)$ is defined as the value embodied in workers' consumption	$e(B)$ is given a priori and workers use the wage to buy commodities at their transformed prices
vd is given a priori	vd is determined after prices of production
r and p are unique (up to scalar multiplication) when:	
d is given	y is given
$r = f(d^*, e(A))$	$r = f(y^*, e(B))$
r varies with the structure of workers' consumption	r varies with the structure of net output
r does not vary with the structure of net output	r does not vary with the structure of workers' consumption
If the numeraire is chosen so that the sum of prices = the sum of values	
Σ profits $\neq \Sigma$ surplus value unless $y = y^*(d)$	Σ wages $\neq \Sigma$ values consumed unless $d = d^*(y)$
but Σ values of the uses of profit $= \Sigma$ surplus value	but Σ profits $= \Sigma$ surplus value

the history of economic thought? I do not think so, because this solution has forced us to explore carefully (as I tried to do in Part II of this paper) the conceptual context of the transformation problem; that is, all those problems connected to the "realisability" of the pair (y, d). In fact, the new solution, precisely because of its simplicity, does not use at all the assumption that the net production y is realised in some balanced model of accumulation. What we gain by expressing r directly as a function of e, and y^*, we lose through the complete indeterminacy of y^*.

Let us take an example (another one is sketched in Appendix C).

Suppose $w(B)$ is the part of the value added which is paid to workers, which they hasten to spend on necessities, and if possible on discretionary items. But, surprisingly, if the structure of production changes, so that the price system changes, as a consequence the frontier of the workers' budget set will move. At a single rate of exploitation, and value of labor power, workers might be able to afford both necessities and some luxuries, or not even their necessities, depending upon the structure of production. This does not fit very well with marxist intuition.

In system A the same phenomenon appears in the following way. Once the value of labor power, $w(A)$ is given, the average rate of profit depends on the structure of workers' consumption, d^*. I do not find this very bothersome.[23] This is simply the guise, transformed and enriched, which the theory of relative surplus value in Volume I of *Capital* takes in the transformation problem. Marx means by relative surplus value the increase in the rate of surplus value which arises from a fall in the value of the goods in the bundle d. But let us suppose a technological change which leaves invariant the value of d, but reduces the technical composition of capital in the sectors producing the elements of d. There is no relative surplus value generated by this change, but the rate of profit, intuitively, ought to rise. This conclusion is made explicit in solution A, which argues that given the value of labor power, the rate of profit will depend on the distribution of workers' consumption among sectors with different compositions of capital.[24]

In contemporary capitalism, characterised by a tight connection—because of the necessity of realising productivity gains—between the substitution of machines for people in production and the spread of higher standards of living to workers (what Gramsci calls "fordism"), the vectors y and d are

[23] This is, of course, a subjective opinion. Dumenil found this conclusion so unacceptable that it moved him to develop solution B.

[24] Here I am mixing up comparative static and dynamic arguments. But we could just as well imagine two goods which are perfect substitutes and have the same value, but are produced in sectors with different technical compositions of capital. If a large number of workers choose one rather than the other, the effect would be the same as that of a technical change of the type I have described.

closely linked by complex dynamic processes[25] which are the foundation of the dialectical play, the loop of Fig. 3. Under these circumstances, it is helpful to be able to call on both solutions A and B.

But we now are raising questions concerning the development of capitalism, which involve the extremely difficult problems of the contradictory pressures on the rate of profit, of crises of realisation, of inflation, and so on. These basic problems Marx and his successors have looked on as having the highest priority, leaving to one side the technical exercise which Samuelson may not have been wrong, in the end, to call the "so-called" transformation problem.[26]

APPENDIX A: The "Tensor of Exploitation"

Marx has analyzed the confusion (common in Smith and Ricardo) between the "value of labor power" and the "value added by labor," and between the "value of a commodity measured by the labor incorporated in it" and the "value of a commodity measured by the quantity of labor (that is, labor power) which it can buy, or command," the two confusions being closely connected. (See K. Marx, *Theories on Surplus Value*, Chapter III.)

It is clear that if we know the duration and the intensity of labor, we can calculate the quantity of labor expended in a day by a unit of purchased labor power m.

Let $\mathscr{m}$ be the n-tuple m_j of quantities of labor power which must be purchased in order to produce one unit of commodity j. This n-tuple is a linear form on the space of commodities, as is the covector ℓ: But instead of measuring the quantities of social labor expended, it measures the quantities of labor power hired by the capitalists. This is not the same linear form as ℓ: They measure different things, and the components of the vectors will also differ (unless we make a special choice of units). But we can transform one into the other by a linear transformation, just as in the study of elastic bodies we can find the vector of strains from the vector of displacements. The mathematical theory of such relations is the theory of tensors, linear operators on vectors and covectors. Here we need a very simple tensor, a "1-covariant, 1-contravariant" tensor T, which can be represented as a matrix:

$$\ell = \mathscr{m}T.$$

[25] These mechanisms are essentially what I call "the monopoly regime of intensive accumulation" (see [9]).

[26] Since the completion of this paper, the new solution has been shown to be easily generalized to production systems which involve fixed capital and rents. This confirms the result that Marx saw as central, that when the numeraire is chosen so that the total price of the net product is equal to the value added, the sum of net profits and rents (and any other revenues of unproductive classes) is equal to the sum of surplus value. See Lipietz [11].

This matrix is the identity matrix I if we choose units appropriately, as I explicitly suggest in the text, and as is assumed implicitly by most students of the transformation problem. In the general case:

$$T_i^i = \varepsilon_i \lambda_i,$$

$$T_j^i = 0 \quad \text{if} \quad i \neq j.$$

This formulation has the advantage of making explicit the two fundamental givens of capitalist exploitation, the duration and the intensity of labor which may eventually be allowed to differ between different sectors (as indeed the wage may vary as well). In the last case, the scalar e ought itself to be replaced by a tensor.

The differentiation of rates of exploitation between sectors which we can express using the tensor representation still would be of limited scientific interest. As Marx recognizes himself, there exist differences in the rate of exploitation, and Engels notes that these differences are likely to be greater than the differences in the rate of profit, since the "equalizing forces are stronger here than there." [9, p. 268]. But two arguments intervene here:

(a) First, taking account of differences in the rate of exploitation misses the point. The crux of the "transformation problem" is that, within the pure labor theory of value, the rate of profit and the rate of surplus value cannot both be equalized across sectors unless the composition of capital is also equal across sectors. But the equality of these two rates is implied by the level of abstraction at which the argument takes place: all the members of one class are equal in their confrontation with the other class (so that the rate of exploitation must be uniform) and in their confrontation with members of their own class (so that wages and profit rates must also be uniform).

(b) If we want a better conceptualisation of concrete reality, we may have to take into account differences between individual members of the two classes. But why privilege sectoral differences in this treatment? The conditions of exploitation in fact probably differ much more by sex, race, region, than by sector.

APPENDIX B: The Mapping that Relates the Rate of Profit to the Rate of Exploitation

(a) *Dumenil's Solution* [6]

Dumenil and Roy compute the inverse function: $e = rv(I - (1+r)A)^{-1} d^*$.

Since d^* is a convex combination of bundles containing only one good, e

lies, for a given value of r, between the extreme points of curves corresponding to these basis bundles. If we trace all these curves (which may intersect) we will have the envelope of the $f(d^*, \cdot)$ functions.

Dumenil also puts forward a more classical expression for f. If y' is the vector of activities which produces net output d (that is, $y' - Ay' = d$), we have $r = e/(1 + (pAy')/(p \cdot d\ell \cdot y'))$, which reproduces Marx's classic formula for the rate of profit, $r = e/(1+q)$, where q is the average organic composition of capital. Here the organic compositions are evaluated at prices of production, and aggregated with the weights given by the vector y'. This result, which is not obvious, allows us to study specifically which curves make up the envelope for a given r.

(b) *Roemer's Solution* [16]

Here is a sketch of Roemer's proof. To a total consumption vector $\mathscr{D}$ satisfying the constraint on w, there corresponds, by the Perron–Frobenius theorem, a system of prices of production p (the wage being taken as the numeraire). To this system p corresponds (through the preference orderings) a total consumption $\mathscr{D}'$. This is reduced by scalar multiplication to a $\mathscr{D}''$ which satisfies the constraint on w. The mapping from $\mathscr{D}$ to $\mathscr{D}''$ is continuous on a compact convex set, so that Brouwer's theorem guarantees the existence of $\mathscr{D} = \mathscr{D}''$. Roemer shows that all the fixed points of this mapping correspond to the same system of prices and the same rate of profit.

APPENDIX C: THE PROBLEM OF NON-BASIC GOODS

Here is another example concerning the indeterminacy of net product in the solutions of type B, which I will not develop here because it relaxes the assumption that the matrix M is indecomposable, and a full discussion would make this paper too long. We know that if there exist non-basic sectors (luxury goods, for example), the rate of profit is determined, according to method A, only by the subsystem of basic commodities. This result bothers some marxists: Why should the average rate of profit not be influenced by the surplus value appropriated in the production of luxury goods in "Department III"? The intuition underlying this question is that if Department III has a very low organic composition of capital, it should be possible to raise the average rate of profit by employing a larger fraction of the labor force in that Department. This intuition depends on the apparent indeterminacy of y in solution B. But this appearance is misleading. The production of Department III must be realised, and leaving aside the consumption of capitalist households which receive the profits of Department III—the output of Department III must be purchased out of the surplus value generated in other Departments. For example, in simple reproduction, the value of the capital enganged in Section III must equal the surplus-value produced in the two fundamental sections (straightforward proof). It turns out that when we take account of this connection, we can reach the same conclusions as in method A. But this gives us directly the conclusion that, once the consumption of workers, d, is given, whatever may be the structure of net output y, given only that it allows for full realization, the rate of profit remains the same. This means that, once workers' consumption is fixed, the net product, whatever assumption we make about the regime of accumulation, is constrained to a region in which profit rate is an invariant determined by solution A, this region being the inverse image of $r=f(d^*, e_A)$ for the function $r=f(\cdot, e_B)$ with $e_B=f(\cdot, d^*, e_A)$. This elegant result ought not to be overlooked.

Acknowledgments

The author wishes to thank Duncan K. Foley for his substantial help in preparing this article and for his useful critical comments. Also he wishes to thank the participants in the CEPREMAP seminar for their comments on an earlier version of the paper.

References

1. M. Aglietta, "A Theory of Capitalist Regulation: The U.S. Experience," New Left Books, London/New York, 1979.
2. J. P. Benassy, R. Boyer, R. M. Gelpi, A. Lipietz, J. Mistral, J. Munoz, and C. Ominami, "Approches de l'inflation: l'exemple français," Publication CEPREMAP, Paris, France, 1977.
3. C. Benetti, C. Berthomieu, and J. Cartelier, "Economie classique, Economie vulgaire," PUG–Maspéro, Grenoble, France, 1975.
4. C. Benetti and Alii, "Marx et l'économie politique. Essais sur les Théories de la plus-value," PUG–Maspéro, Grenoble, France, 1977.
5. G. Dostaler, "Valeur et Prix. Histoire d'un débat," PUG–Maspéro, Presses de l'Université de Québec, Montréal, Canada, 1978.
6. G. Dumenil and C. Roy, Pour une approche fonctionnelle du théorème marxien fondamental d'Okishio–Morishima, *Cahiers d'économie politique*, n° 7, P.U.F., Paris, 1982.
7. G. Dumenil, "De la valeur aux prix de production," Economica, 1980.
8. D. Foley, On the transformation of surplus value into profit, unpublished, 1979.
9. A. Lipietz, "Crise et inflation: Pourquoi?," F. Maspéro, Paris, 1979.
10. A. Lipietz, Conflits de répartition et changement technique dans la théorie marxiste, *Econ. Appl.* **2** (1980).
11. A. Lipietz, Nouvelle solution au problème de la transformation: le cas du capital fixe et de la rente, *Recherches économiques de Louvain*, n° 4, 1979.
12. K. Marx, "Capital," International Publishers, New York, 1967.
13. M. Morishima, "Marx's Economics: A Dual Theory of Value and Growth," Cambridge Univ. Press, Cambridge, 1973.

14. M. MORISHIMA AND G. CATEPHORES, "Value, Exploitation and Growth," McGraw–Hill, London, 1978.
15. H. NIKAIDO, "Introduction to Sets and Mapping in Modern Economics," North-Holland–American Elsevier, Amsterdam/New York, 1970.
16. J. ROEMER, Marxian models of reproduction and accumulation, *Cambridge J. Econ.* **2** (1978), 37–53.
17. P. SALAMA, "Sur la valeur. Eléments pour une critique," Maspéro, Paris, 1975.
18. P. A. SAMUELSON, Understanding the Marxian notion of exploitation: A summary of the so-called transformation problem between Marxian values and competitive prices, *J. Econ. Lit.* **9** (1971), 399–431.
19. D. YAFFE, Valeur et prix de production dans "Le Capital" de Marx, *Critiques Econ. Politique* **20** (1975), 45–103.

Printed by the St. Catherine Press Ltd., Tempelhof 41, Bruges, Belgium