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The So-Called "Transformation Problem" Revisited

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Much ink has been spilled since von Bortkiewicz first criticized the way Marx dealt with the problem of transforming values into prices of production in Volume III of *Capital* [12]. Although no one any longer pretends that this discovery marks the beginning of the "crisis of marxism" (so technical a question deserves neither the credit nor the blame for that), it remains true that the solution most widely accepted at the moment (which is expressed most completely in the work of M. Morishima[1]) greatly weakens the thrust of Marx's theory of value and surplus value. The accepted approach reduces that theory at the most to a rough and approximate expression of the idea that workers do not receive the whole fruits of their labor in the wage. This at any rate is the conclusion of Morishima himself in his "Fundamental Marxian Theorem" ("the equilibrium rate of profit is positive if and only if the rate of exploitation is positive"), and of Samuelson, who says

> Although *Capital*'s total findings need not have been developed in dependence upon Volume I's digression into surplus values, its essential insight does depend crucially on comparison of the subsistence goods needed to produce and reproduce labor with what the undiluted labor theory of value calculates to be the amount of goods producible for all classes in view of the embodied labor requirements of the goods. The tools of bourgeois analysis could have been used to discover and expound this notion of exploitation if only those economists had been motivated to use the tools for this purpose. [18, p. 421]

What Samuelson means by "tools of bourgeois analysis" is the theory that the time by which an owner of money agrees to defer his consumption is as scarce a good as labor, and deserves its own special income (interest).

* Translated into English by Duncan K. Foley with the assistance of the author.

[1] See especially his "Marx's Economics" [13]. Earlier work in this line includes that of Medio, Meek, Okishio, Seton, Sweezy, Roubine, Winternitz, and others. All those mentioned treat the question along the same general lines as Morishima. For a survey of this literature, from Marx to von Bortkiewicz, see Dostaler [5]; from von Bortkiewicz to the present, see Samuelson [18] and Benetti *et al.* [3]. Morishima and Catephores have proposed a new solution in terms of Markov processes [14]. But this work adopts the same approach as the earlier work on the essential question which will be developed here, the treatment of variable capital.

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In this situation several Marxist writers take refuge in a refusal to adopt formalisations of Morishima's kind, putting forward critical observations which are coherent, but do not address this issue directly (Salama [17] and Yaffé [18]) or even refuse, in the name of doubtful epistemological principles, to confront the problem of the transformation at all (Benetti *et al.* [3]).

I propose to argue here that solutions of Morishima's kind, once they are filled out and correctly understood, do not contradict any of Marx's aims in *Capital*, but that there exists another solution,[2] closer to the approach of *Capital*, which exhibits the famous equalities of Volume III, between the sum of prices and the sum of values, and the sum of profits and the sum of surplus value, equalities which are inconsistent with Morishima's treatment. Moreover, it will be shown that the rate of profit in this treatment depends on the rate of exploitation and the structure of output, rather than on the workers' consumption bundle, as in Morishima's analysis.

The first part of the paper will review Marx's treatment of the problem, and the critiques of it; the second part will treat Morishima's solution; the third will put forward the new solution; and the last part will compare the properties of the two solutions.

I. Marx's Solution and Its Limitations

For Marx,[3] the commodity character of the economy confers on the products of economic agents a value proportional to the fraction of the social labor allocated to their production and validated by society in exchange. To adopt modern terminology, one could say that to each bundle of commodities, represented by a vector $\mathscr{y}$ in the space having for its natural basis the n different units of use-value, value, a linear form $\mathscr{v}$ on that space, associates a positive number.[4]

In an assumed representative productive process, the "living" labor applied to means of production adds value to the value already created by

[2] Dumenil [7] laid the foundations of this solution. I would like to thank him for helpful conversations on this problem, though my approach is rather different from his. At the same time, but completely independently, Foley [8] proposed a substantially similar solution.

[3] The extremely brief summary which follows raises many problems which are discussed in relation to Marx's text in my book [9]. Here I confine myself to an examination of the formalism used in studying the "transformation problem."

[4] Vectors $\mathscr{y}$ (and linear forms, or covectors $\mathscr{v}$) will be denoted by script and are treated as column vectors in right-hand side operations (and row vectors in left-hand side operations). According to the Einstein convention, indices are superscript for vectors (and subscript for covectors). Scalars and matrices will be denoted by italics.

"dead" labor incorporated in the means of production.[5] Then, if a^i_j is the quantity of good i normally required to produce one unit of good j, and if A is the corresponding matrix of these production coefficients, we have $v = vA + \ell$, where $\ell = l_1, \ldots, l_n$ is the vector of quantities of abstract labor incorporated concretely in units of the goods. As a result:

$$v = \ell(I - A)^{-1}.$$

This procedure of "adding" current labor to the value already embodied in the goods[6] has been criticised, and some French marxists refuse to adopt it (see [4, Sect. 24]). We can show nevertheless that it is legitimate, and inherent in the form of value, as long as the techniques of production remain the same, and we are analyzing the reproduction of the economic system. Under these assumptions the past labor expended in one branch of production corresponds to the current labor expended in another branch. Of course, this no longer holds if we permit changes in technique, which are part of the causes of crisis and inflation (see [9]). In any case, the debate on the "transformation problem" always (though rarely explicitly) makes these assumptions.

So far we have had nothing to say about the capitalist character of the economy. The marxist approach, like all scientific approaches, proceeds by successive approximations: We first study bodies falling in a vacuum, then introduce the resistance of the atmosphere, magnetic fields, etc. In the first section of Volume I of *Capital*, Marx enunciates and analyzes the law of value (its substance, form, quantity) "in general," that is, for any commodity economy. He then analyzes the "divisions" of the flow of value which correspond to capitalist social relations (between constant capital C, variable capital V, and surplus value S), and finally, in Volume III, the modifications induced in the law of value itself by those relations.

In capitalist social relations labor power appears (to the capitalist, not to the worker, of course) as a commodity. This commodity has a value w: the amount of abstract labor which workers have the "right," given the historical stage of development, to spend on the market to reproduce their labor power from day to day. It also has a use value: the ability to produce abstract labor, and hence to add value to commodities. The amount of abstract labor

[5] By "representative productive process" I mean a process operating with current standard techniques. Such a process is characterized by a technique which gives the quantities of means of production and labor required, and the time of labor taken by the technique. See Aglietta [1] and Lipietz [9].

[6] Notice that v and ℓ are measured in the same units, abstract labor time. Once v is computed, we could renormalize the matrix A by choosing as the unit of each good the quantity of the good which has unit value. The coefficients a^i_j would then be pure numbers, rather than quantities of good i per unit of good j.

va (value added) that can be extracted from this commodity is determined by the duration, λ, and the intensity, ε, of labor. The three parameters w, λ, ε are the results of a historical process, of the "class struggle" (see Marx [12, Vol. I, Chap. X]). Together they determine the rate of surplus value, or rate of exploitation, e, which is the ratio of the "unpaid labor," $va - w$, to the value of the labor power w. Thus by definition:

$$w(1 + e) = 1.$$

In modern vector terminology, we have

$$v = vA + w\ell + ew\ell.$$

This is the modern form of "$C + V + S$." Notice that in adopting this notation we have assumed that the quantity of the commodity labor power which it is necessary to buy and put in motion to produce the good j is measured by the same number as the quantity of abstract labor produced by that labor power. That is, we have assumed that the duration and intensity of work are given, and that we have taken as our unit of labor power the day (for example), and for our unit of value the abstract labor expended in a day. The techniques of production, of course, are taken to be fixed socially, and concrete labor is counted directly as social abstract labor. To put it another way, we assume the existence of a linear transformation T, mapping n-tuples of the commodity "labor power" into covectors of "value added." This transformation, the coefficients of which are defined by the intensity and duration of work, could be called the "tensor of exploitation" (see Appendix A). These may appear to be relatively minor points, but if we forget them we are in danger of identifying, along with the pre-Marxist classical economists (Smith, Ricardo), "labor commanded" and "labor embodied,"[7] and, what is very important for our discussion, of confusing a social relation with a technical relation between certain input–output coefficients.

The illusion of a purely technical relation between inputs and outputs is completed once we suppose, as we are entitled to do, that, just as the production of good j is characterized by the given coefficients of the representative process (a_j^i, l_j), so there exists a vector of normal consumption per unit of labor time, d. We then have

$$w = v \cdot d$$

[7] The Benetti and Cartelier criticism [3] of Morishima-type solutions turns on this point. Still, instead of positing the existence of this tensor, these writers refuse point blank to confront the problem of the relation between the space of values and the space of relative prices.

and the existence of the autonomous commodity "labor power" necessary to the production of a unit of commodity j, l_j, is absorbed by the given bundle of commodities necessary indirectly for the production of a unit of j, the vector $(d^i l_j)$, which must be added to the vector (a^i_j). Then we get a "social–technical" matrix by adding to A the tensor product (or Kronecker product) $d \times \ell$:

$$M = A + d \times \ell,$$

which is only an abbreviation for:

$$M = A + d \times (\ell T^{-1}),$$

the linear transformation T being reduced to the identity matrix by the choice of units. This matrix looks purely "technical," but the three determinant elements of the labor theory of value and exploitation are already incorporated in it. d expresses the value of labor power, once it is multiplied by v; and T the relation between the quantity of labor power and the quantity of labor embodied in the commodities (which depends on the duration and intensity of labor).

But this is only the beginning of the development of the law of value. If the exchange of products is regulated through competition on the average by the value of products in a pure exchange economy, what regulates exchanges between products of economic agents which are capitalists, in an economy where "commodities are exchanged as products of capital" [12, Vol. III, p. 175], that is, products of labor hired by capital, not of labor alone? The answer, according to Marx, is a transformed value, in which the quantity of surplus value (now called profit) received by each capitalist is proportional to the capital invested. The value produced in a period by society as a conserved quantity finds itself reallocated (again, by competition, but by the competition of capitals [12, Vol. III, p. 180]) among the products, the "transformed" values, and their expression in money, the "prices of production," being fixed so that the surplus value (that is, the part of the produced value which does not return to the direct producers) is distributed among the capitalists in proportion to the capital they have invested. As a result the sum of the prices of production will be equal to the sum of the values, and the sum of the profits will be equal to the sum of surplus value, or at least the ratio of these two pairs of quantities will be the same (the ratio depending on the choice of the numeraire).

Is this mathematically possible? Marx thought so, since this result is a direct expression of his theory of value and of exploitation. And he devised in Volume III of *Capital* (of which he left only a draft at his death) a simple algorithm.

Suppose that we divide the economy into branches, each producing a value: $M_i = C_i + V_i + S_i$. (In the earlier notation, M represents the quantity of value $v_i y^i$, where y_i is the quantity of the good produced in sector i.) In each branch the capital invested has a value $C_i + V_i$ (assuming that the rate of turnover is unity). The total capital engaged is $\Sigma(C_i + V_i)$. The total surplus value is ΣS_i. The general rate of profit is $\Sigma(S_i)/\Sigma(C_i + V_i) = r$. If every branch must realise the same rate of profit, then we need only redistribute the surplus value, and we have the price of production in each sector:

$$PP_i = (C_i + V_i)(1 + r).$$

Clearly the sum of these prices of production will be equal to the sum of the values, and the sum of the profits at these prices will be the same as the sum of the surplus value. The average rate of profit is determined by the rate of surplus value, e; the organic composition of capital in the various sectors, C_i/V_i; and the distribution of the total social capital in these different sectors, and hence by the vector of outputs, $\mathscr{y}$ [12, Vol. III, p. 163].

This model has two limitations. First, it assumes that all the sectors have the same period of production. Marx worked out long calculations to investigate the effect of weakening this assumption. But that assumption is not the focus of the controversy. In theory it is always legitimate to bring in complications one by one. We will proceed this way ourselves, in this article.[8] The discussion has centered on another assumption, this one less defensible. Capitalists do not buy the elements of constant capital and variable capital at their values, but at their prices of production. The redistribution of value affects not only S, but also C and V. Marx himself notes this, (e.g., [12, Vol. III, p. 165]), but judges that "our present analysis does not necessitate a closer examination of this point," and passes on to subjects which he judges to be more important. What a mistake! People won't forgive the small slips of great men. And, worse yet, those who have undertaken to correct this slip have proceeded in a path which denies the basic point of Marx's argument: that profit, far from being a compensation for waiting paid to voluntary saving, is only an unpaid part of the value created by the productive workers.

II. The Accepted Solution to the Transformation Problem

To correct Marx's slip, we must assume that the elements of constant and variable capital are bought at their prices of production. For the elements of

[8] On the introduction of fixed capital, see [13] and many other sources. In the same spirit, I will always assume that the matrices are indecomposable.

constant capital this is not a great difficulty: It is necessary only to evaluate the inputs at their prices of production. But what does it mean to say that variable capital is purchased at its price of production? This is a complex issue on its face. Variable capital is, properly speaking, only a quantity of money paid to workers, which represents a fraction (determined by the rate of exploitation) of the value added. Those who first put forward solutions to the problem hit upon an idea that, as it turns out, is not at all neutral. They identified the value of labor power, not with this fraction of the value added, but with the value of the goods d which the money would buy if all the workers adopted the same pattern of consumption and spent their money in a market where prices were proportional to the system of labor values.

Let us follow this path for a moment. As a consequence, we must treat variable capital like constant capital, and evaluate the bundle d at the same prices of productions. Suppose p is the covector of prices, and r the average rate of profit, if it exists. Then:

$$p = (pA + (p \cdot d)\ell)(1 + r),$$
$$(1/(1 + r))\, p = p(A + d \times \ell) = pM.$$

p is thus the eigenvector corresponding to the eigenvalue $(1/(1 + r))$ of the social–technical matrix M.

p is semi-positive, as is M. The Perron–Frobenius theorem (see Nikaido, [15]) tells us that p must be the eigenvector associated with the largest eigenvalue $\mu(M)$. We have, then,

$$r = 1/\mu(M) - 1.$$

First of all, notice that μ depends only on M, hence on A, ℓ, and d, which also determine e. Among the set of vectors d such that $v \cdot d = w$ (that is, for a constant rate of surplus value), the rate of profit r depends on the composition of d, that is, on workers' consumption, and not at all on the structure of output y, that is, not on the "distribution of capital in the different spheres."

But there is worse to come. Suppose we choose the numeraire so that the sum of prices equals the sum of values, that is, $v \cdot y = p \cdot y$. We have that the sum of profits is $rpMy$, and the sum of surplus value is $ew\ell y$. These sums are equal only if

$$(rpM - ew\ell)y = 0,$$

a relation which there is no reason a priori to assume. As a result, except on a set of zero measure of possible structures of production, we cannot have:

$$\frac{\text{sum of prices}}{\text{sum of values}} = \frac{\text{sum of profits}}{\text{sum of surplus value}}.$$

These two results, that the rate of profit is independent of the structure of production, and that value and surplus value are not conserved in the transformation, many Marxists prefer not to face. In my opinion, they are wrong in this, not only because Morishima and Samuelson offer a drop of comfort in the form of the "fundamental Marxian theorem," that the rate of profit is positive if and only if the rate of exploitation is also positive.[9] Marxists can face this situation because in fact it is possible within this conceptual framework to draw almost all the conclusions Marx tried to prove in his model.

1. *The Value of Commodities Recovered by Capitalists Is in Fact the Surplus Value*

In other words, even if the sum of profits is not the sum of surplus value, the value of the uses of profits is certainly the sum of surplus value. The proof of this theorem is quite trivial. It is enough to notice that in writing

$$p = (1 + r)\, pM,$$

we implicitly assume that the whole product is realized in sale, that there is no overproduction.

But the gross product $\mathscr{Y}$ produced in a period serves: to replace the inputs used up, $M\mathscr{Y}$; for the unproductive consumption of the capitalists, $\mathscr{C}$; and for the production of the elements of the expansion of production, or accumulation, $M\Delta\mathscr{Y}$. These two last terms constitute the use of the profit (the first term representing the uses of the initial capital). We have

$$\mathscr{Y} = M\mathscr{Y} + \mathscr{C} + M\Delta\mathscr{Y},$$

$$v(\mathscr{Y} - M\mathscr{Y}) = v(\mathscr{C} + M\Delta\mathscr{Y}),$$

$$ew\ell\mathscr{Y} = v(\mathscr{C} + M\Delta\mathscr{Y}),$$

which establishes the claim.

Intuitively we can see that if the prices at which capitalists sell the commodities differ from the values, the same thing happens with the commodities they buy, and the one difference balances out the other, in a sense which we must now make explicit.

2. *The Prices of Production Regulate the Behavior of Capitalist as "the Personification of Capital"*

In the last paragraph, the "unproductive consumption" of the capitalists was indeterminate, and it could not be otherwise at this level of abstraction. $\mathscr{C}$ could be determined only by considerations of a social–psychological

[9] See [13, 18]. We will present later a much stronger result.

nature. At this level, the capitalist is only, as Marx says, the "personification of his own capital," value which has only one goal, to expand. If then we reduce the capitalist to his essence, he would obey the famous protestant ethic of Max Weber, and the "duty to society" of Calvin: Frugal in the extreme, he would accumulate all his own capital, and invest it in his own sector, since there is no reason (at this level of abstraction) for him to do anything else.

Let then $p_i y^i(t)$ be the revenue of sector i in period t; it is completely used for the purchase of inputs to production in $y^i(t+1)$:

$$\begin{aligned} p_i y^i(t) &= (pM)_i\, y^i(t+1) \\ &= (1/(1+r))\, p_i y^i(t+1). \end{aligned}$$

Then

$$y^i(t+1) = (1+r)\, y^i(t).$$

On the other hand, these inputs to production (which include the subsistence goods for the newly recruited workers) represent the whole product of period t:

$$y(t) = My(t+1)$$

so that:

$$y(t+1) = (1+r)\, My(t+1).$$

The vector of production thus must be the righthand eigenvector y^* of the matrix M corresponding to the dominant eigenvalue, and grows exponentially at rate $1+r$. This is the famous growth path which Morishima calls the "Marx–von Neumann ray," and which I will call the "structure of integral accumulation," and which allows maximal balanced growth.

For this structure of production (which is of measure zero in the space of possible structures, but plays an undeniably special role) it is easy to show that, if we choose the covector of prices of production so that $p^* \cdot y^* = v \cdot y^*$, then we will have the sum of profits is equal to the sum of surplus value. In fact,

$$\text{sum of profits: } rp^*My^* = (r/(1+r))\, p^* \cdot y^* = (r/(1+r))\, v \cdot y^*,$$

$$\text{sum of surplus value: } v \cdot y^* - vMy^* = (r/(1+r))\, v \cdot y^*.$$

3. *A Compensation Theorem for Values and Prices*

These two reference systems, p^* and y^*, will let us make rigorous the intuition, stated in paragraph 1, of a necessary compensation of the divergence between values and prices of production by the structure of production. Any product vector y can in fact be decomposed uniquely into a component y^* parallel to the integral structure of accumulation, and a component y' (which may have negative components) in the hyperplane orthogonal to the price system p^*:

$$y = y^* + y', \qquad \text{where} \quad p^* \cdot y^* = v \cdot y^* \text{ and } p^* \cdot y' = 0.$$

If we write

$$\delta v = p^* - v = \text{divergence of } p^* \text{ from } v,$$
$$\delta y = y - y^* = \text{divergence of } y \text{ from } y^*,$$

then

$$\delta v \cdot y + v \cdot \delta y = 0.$$

In fact,

$$\begin{aligned}\delta v \cdot y &= (p^* - v) \cdot y = p^* \cdot y^* + p^* \cdot y' - v \cdot y^* - v \cdot y' \\ &= -v \cdot y' = -v \cdot \delta y.\end{aligned}$$

To put it another way, if, starting from a path of reproduction corresponding to the structure of integral accumulation (and choosing the numeraire so that price is equal to value in total) we "deform" the structure of production in a certain direction, the measure of that deformation of production (in terms of values) is the measure of the difference between total price of production (and total value) for that structure of production. In this way we find a relation (certainly much weaker than Marx's claim) between the structure of production and the transformation of values into prices of production. But p^* and y^* remain functions of d, so that y is only a way of evaluating after the fact their relative variations.

4. *The Rate of Profit Is a Well-Defined Function of the Rate of Exploitation*

If one is not happy with the "fundamental Marxian theorem," one can try to express explicitly the relation between e and r (given the parameters A, d, and ℓ). But a function of the form $r = f(d, e)$ is subject to the objection that d and e are related, since $(1 + e)\, v \cdot d = 1$. We must separate more clearly the rate of exploitation from the bundle of goods workers consume. Two suggestions have been made as to how to do this.

The method of Dumenil and Roy [6] (see Appendix B) is to determine the workers' consumption bundle d in two steps, one dealing with the composition of the bundle, the other with the rate of exploitation. He writes:

$$d = wd^*,$$
$$v \cdot d^* = 1,$$

to express the idea that the workers choose a bundle of consumption goods from the simplex of bundles whose value is unity, and then scale this bundle to the rate of exploitation. The curves $r = f(d^*, e)$ are then defined by a simple relation; they are all convex, monotonically increasing in e, and bounded by R, the dominant eigenvalue of the matrix A of technical coefficients of production (which coincides with M when workers "live on thin air"). The envelope of this family of curves is bounded by the functions which correspond, for a given value of r, to consumption bundles d^* which maximise and minimise the organic composition of capital defined in a particular way.

Another, more complicated method has been proposed by Roemer [16] (see Appendix B). It allows workers to choose their consumption bundles individually. Each worker is assumed to have a preference ordering γ (with the usual neoclassical properties of convexity and continuity). Given the wage and the prices of production, each worker chooses freely how to spend his income. By using the Perron–Frobenius Theorem in conjunction with Brouwer's Fixed Point Theorem, Roemer shows that there exists a system of prices at which workers will choose bundles of goods which each have a value w. The curves are now indexed by the family of preference functions of the workers, Γ, but the shape of the family of curves $r = f(\Gamma, e)$ is the same.

Nevertheless, the use of Brouwer's theorem (which is an abstract result without direct economic relevance) raises a fundamental problem for this arguement. How could workers choose their bundle of consumption goods facing prices of production when their budget constraint is fixed in the system of values by the requirement $v \cdot d = w$? This problem is related to the very doubtful procedure which is common to all the Morishima-type solutions, the transformation of variable capital V on the basis of a given

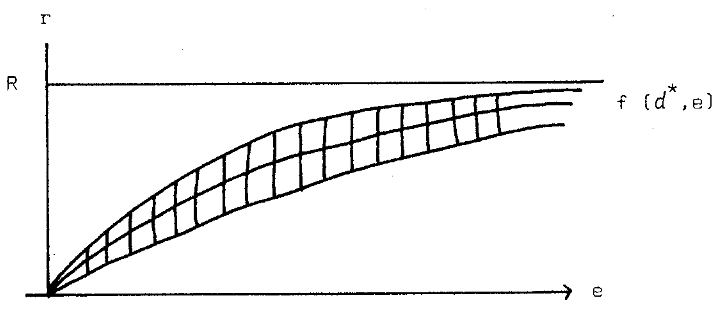

FIG. 1. Rate of exploitation and rate of profit.

volume of workers' consumption. To repeat the obvious, workers receive money, not a voucher for a bundle of commodities. Once the transformation has been achieved, and choices are made in the price system, the relation $r=f(d^*, e)$ is certainly true. But does it correspond to a causal relation between d^* and e on the one hand and r on the other? Certainly for Marx the causal relation was between y and e on the one hand and r on the other. Roemer's and Dumenil's contributions do not eliminate this weakness in the accepted solution to the transformation problem. We will return to this problem below.

5. *The Prices of Production and the Rate of Profit Are Logically Determined by the Labor Theory of Value and of Exploitation*

We have arrived at a critical point in the controversy. However strong may be the links between the system of values and surplus value on the one hand, and the system of prices and profit on the other, as we have just shown, these connections appear at first sight to be due to the fact that both systems depend on a third set of data, the given "technique" as expressed by the matrix $M=A+\dot{d}\times\ell$.

In fact, the matrix M determines a "surplus," given only the technical condition that it is "viable," that is, produces more output than its inputs, and this net product is allocated according to two different principles: (a) in proportion to the labor expended during the last period on each commodity in the value system; (b) in proportion to the total costs of production (including both raw materials and labor) incurred in the last period.

Samuelson [18] compares these two schemes to the effects on the price system of a value-added tax on the one hand, and a turnover tax on the other. In the first case the solution of the system of equations determining prices is extremely simple, being a system of n simultaneous linear equations. In the second case we have to solve a polynomial equation of the nth degree (namely, $\det[I\mu - M]=0$). Samuelson suggests:

> One might apply Marx's theory of the materialist determination of history to arrive at the hypothesis that it was Marx's incapacity in algebra and the absence of a

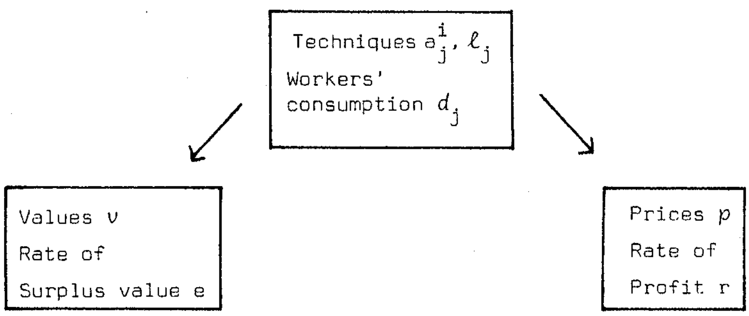

FIG. 2. The apparent common ancestry of values and prices.

computer that caused him to formulate his exploitation theory in Volume I terms which are unrealistic but which happen to be simpler to handle algebraically than Volume III's Walrasian relations. [18, p. 418]

The transformation problem can then be solved very easily, according to Samuelson (always on the basis of the given "technical" conditions A, d, and ℓ):

The "transformation algorithm" is precisely of the following form: "Contemplate two alternative and discordant systems. Write down one. Now transform by taking an eraser and rubbing it out. Then fill in the other one. Voila. You have completed your transformation algorithm." [18, p. 400]

Unfortunately, this pretty story assumes that the matrix M, from which are deduced, it is true, the price system p, and r, is given logically prior to the labor theory of value and exploitation. But we have already seen that this is not the case.

The idea that the "natural" or "technical" productiveness of the matrix M is the origin of surplus is an old idea, which Marx criticized when he found it in the post-Ricardians, especially J. S. Mill.

Favourable natural conditions alone give us only the possibility, never the reality, of surplus labour, nor, consequently, of surplus-value and of surplus product. These conditions affect surplus-labour only as natural limits, i.e., by fixing the points at which labour for others can begin. In proportion as industry advances, these natural limits recede. In the midst of our West European Society, where the labourer purchases the right to work for his own livelihood only by paying for it in surplus labour, the idea easily takes root that it is an inherent quantity of human labour to furnish a surplus-product. [12, Volume I, chapter XVI, p. 515].

Marx illustrates this point by describing a Pacific people who can meet their needs by working one day a week, until as a result of colonization they are forced to work the whole week.

How do the given techniques of production in the algebraic model conceal the social relations? Let us go over the logical chain step by step.

First we have the commodity character of the economy. From this we develop the substance and the form of value. For a given state of the productive forces (A, ℓ), we can derive the magnitudes v of the vector of values. Up to this point there is not a word about exploitation, nor about surplus value, nor about profit. Remember that ℓ expresses the quantities of abstract labor necessary to the production of the goods. These quantities might correspond to ℓ, 2ℓ, 3ℓ days of wage labor, depending on the intensity and duration of work.

We introduce capitalist exploitation, the sale and the use of the commodity "labor-power," characterised by the value of labor power and the duration λ and the intensity ε of work. The value of labor power, w, corresponds (through v, which we have defined already) to a bundle of goods, d, necessary to reproduce the labor power (at least this is how the Morishima-type argument proceeds.)

All these elements, λ, ε, w, d, and e, are directly or indirectly the objects of class struggle, and are related to each other in complicated ways.

λ and ε are weakly related to each other, and even more distantly connected to d. (We know that production falls only 1% when the length of the working day falls by 2%, for example). We can therefore consider λ and ε logically prior to e and to the amount of surplus value. They are implied in the definition of the linear transformation which maps covectors of the quantity "labor power" into covectors of "value added." This mapping, which is an identity when the units are properly chosen, is necessary to link the "technical" data to the equations defining prices of production:

$$p = (1 + r)(pA + w\ell) \qquad \text{(where } w = \text{wage rate)}$$

being only an abbreviation for:

$$p = (1 + r)(pA + w\ell T^{-1}).$$

λ and ε being given, however, the relation between e, d, and w is much more complicated. The dynamic process that appears at first glance to be playing the decisive role is from e to w to d. That is, capitalists and workers confront each other over the division of the value added, the result defining the value of labor power w, which workers spend in money as they wish. Hence, this defines, through the system of values or prices, the bundle of commodities d which the workers can eventually purchase.

Nevertheless, Marx, in Chapter VI of *Capital*, anxious to give an intuitive sense to the formulation that "labor power, like any commodity, has a value, namely, the quantity of labor necessary to its reproduction" stretches the analogy to some degree in supposing the existence of a bundle of workers' consumption, a kind of vector input to the household-enterprise (whose worker, let us remark in passing, the wife, works for free, and whose proprietor sells the product at its cost). Leontieff, von Neumann, and Morishima codify this reduction of the worker to a beast of burden which needs its feed.

Although Marx quickly abandons this physical determination of a bundle of wage goods, and studies the value of labor power as a "quantity of paid labor,"[10] his assumption that there exists at any given moment a "standard of worker's consumption" is not indefensible. Certainly unions do not bargain directly for washing machines or color televion sets; to repeat yet

[10] Samuelson quite freely admits this [18, p. 422], so that it is surprising that he accepts Seton's solution (which depends on the concept of a physical bundle) as a solution to Marx's transformation problem.

$$\begin{array}{ccccccc} & & e & \rightarrow & w & & \\ & & \uparrow & & \downarrow & & \\ v \text{ or } p & \rightarrow & w & \leftarrow & d & \leftarrow & v \text{ or } p \end{array}$$

FIG. 3. The "loop" $e - d$.

again, they bargain over increases in wages. Nevertheless, a "standard of living" once it is widely adopted, can be lowered only with difficulty, not for moral reasons, but because of the vested interest of the sectors which produce those consumption goods.[11] So there is a feed-back from the historical standard of living (given by the bundle d) to the value w (and hence to e) through the system of prices or values.

We could say that e and d are "dialectically" related, d being the base, and e the directing factor. The logical chain looks like Fig. 3.

Once we have fixed d, and required that wage cover the cost of d at the prices of production, we can move to the last link in the logical chain, summed up in Fig. 4. The striking comparison with Fig. 2 justifies the careful way we have uncovered the meaning of the algebraic symbolism used in the accepted transformation. Even if we take d as given, the labor theory of value and of exploitation appear clearly as logically prior to the analysis of prices of production.

III. A NEW SOLUTION TO THE TRANSFORMATION PROBLEM

The usually accepted solution to the transformation problem, then, has now been shown to be less inconsistent with Marx's theses than many scholars think. Nevertheless, it still deviates from Marx's ideas in significant ways.

(a) If the labor theory of value and of exploitation is logically prerequisite to the calculation of prices of production, this fact is not clearly revealed in the usual algorithm of transformation. One would like to "see" the value being redistributed over the commodities in the process of equalization of rates of profit.

(b) If there exists a relation between the structure of output and the deviation of prices from values, it appears "ex-post" in the usual treatments of the problem. But in Marx's treatment the structure of output (which determines, through the weights applied to the different organic compositions

[11] This is a critical point in understanding the inflationary form of the current crisis. See [1, 2, 9].

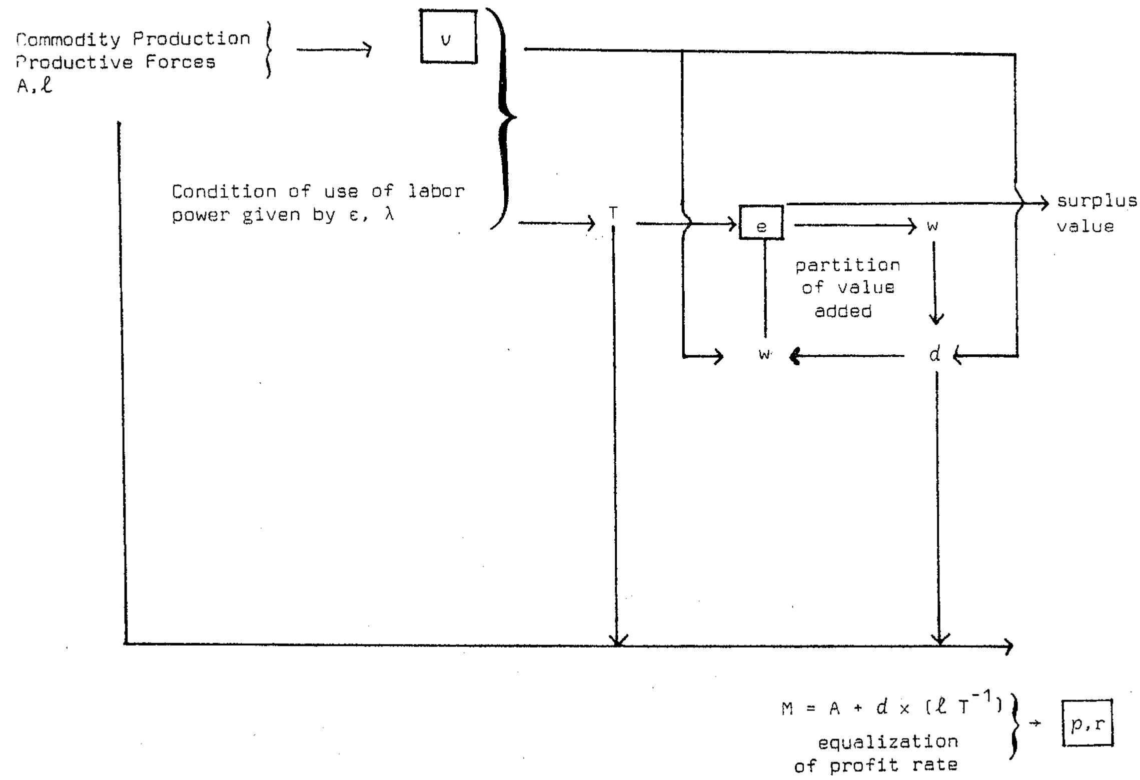

FIG. 4. The conceptual filiation from value and exploitation to prices and profits.

of capital, the quantity of surplus value to be distributed among the invested capitals) appears as a determining parameter for the rate of profit and prices of production. In the usual treatment the structure of workers' consumption seems to play this determining role.

1. *The Root of the Question*[12]

We have seen that all the anomalies between Marx's intuitions and the Morishima-type solutions have their origin in the way Bortkiewicz and those who follow in his tradition have treated the problem of the transformation of the value of labor power. They all treat V (or in the present notation, w) exactly like constant capital, C. No one would deny that if the technique of production, (a_j^i, l_j) is given[13] the cost price of constant capital C is $\Sigma p_i a_j^i$.

[12] The credit for the discovery of this new solution belongs chiefly to Dumenil [7], who identified the two points necessary to carry it out: the interpretation of the value of labor power, w, as a share of value added; and the application of the transformation to the net product. His arguments rest on a deep understanding of the conceptual framework of *Capital.* The responsibility for the present exposition, which follows my own analysis in [9], and for the arguments in the next paragraphs, is mine.

[13] Of course, the least costly technique could depend on e and on r. Even worse, a technique could be the cheapest in value, but not the cheapest in prices of production. Here we will ignore these dynamic problems (see [10]).